

SWOT INSTITUTE
INVERSE TRIGONOMETRIC FUNCTIONS
XII-TEST

Time : 1 hr.

1. Find the value :

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right).$$

2. Show that $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$.

3. Express $\tan^{-1} \frac{\cos x}{1 - \sin x}$, $-\frac{3\pi}{11} < x < \frac{\pi}{2}$ in the simplest form.

4. Prove : $2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$.

Write the following functions (Q. No. 5 to 7) in the simplest form.

5. $\tan^{-1}\left(\sqrt{\frac{1 - \cos x}{1 + \cos x}}\right)$, $x < \pi$.

6. $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$

7. $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$, $a > 0$; $-\frac{a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$.

Find the value (Q. No. 8 to 9)

8. If $\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x .

9. If $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x .

10. Show that $\sin^{-1} \frac{3}{5} - \sin^{-1} \frac{8}{17} = \cos^{-1} \frac{84}{85}$.

11. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

12. Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}$, $x \in \left(0, \frac{\pi}{4}\right)$

13. $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2} \cos^{-1} x$, $-\frac{1}{\sqrt{2}} \leq x \leq 1$ [Hint : put $x = \cos 2\theta$]

14. Solve : $2 \tan^{-1}(\cos x) = \tan^{-1}(2 \operatorname{cosec} x)$